

The Global Positioning System (GPS) and Attitude Determination: Applications and Activities in the Flight Dynamics Division

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Abstract

The application of GPS to spacecraft attitude determination is a new and growing field. Although the theoretical literature is extensive, space flight testing is currently sparse and inadequate. As an operations organization, the Flight Dynamics Division (FDD) has the responsibility to investigate this new technology, and determine how best to implement the innovation to provide adequate support for future missions.

This paper presents some of the current efforts within FDD with regard to GPS attitude determination. This effort specifically addresses institutional capabilities to accommodate a new type of sensor, critically evaluating the literature for recent advancements, and in examining some available -albeit crude- flight data.

pair. The method is more commonly referred to as interferometric measurement, and has been employed before in ground based receivers for the purpose of tracking a spacecraft's position.

The interferometric principle involves a passive system comprised of two antennas, separated by some baseline, receiving a signal from the same source. Antennas onboard an orbiting spacecraft uses the signals received from the individual GPS satellites as sources. From this information the direction cosine between the baseline and the line of sight to the GPS spacecraft is determined. With the use of another baseline, preferably orthogonal to the first, the direction cosines between the line of sight from that baseline to the same GPS spacecraft is obtained. Finally, the direction cosine of the third axis, orthogonal to the other baselines, is known. From these direction cosines, a unit vector to a known point in space is determined and is analogous to the use of data from typical attitude sensors for attitude determination.

Background

Originally the constellation of GPS spacecraft, currently numbering 24, was conceived to produce accurate position and time information for ground, air, and space based systems. Although the accuracy is degraded, this information would be available to anyone with a GPS receiver on a continuous basis. In addition, it was later discovered that with a pair of GPS antennas a user can determine a phase difference between signals of the antennas and consequently attitude. This phase difference is related to the angle between the line of sight to the GPS satellite and the baseline connecting the antenna

The first full test using GPS data and a star tracker attitude truth (better than 1 arc min) for attitude determination and control onboard the spacecraft will be on a Spartan spacecraft (the GPS Attitude Determination And Control System, or GADACS) to be launched in the fall of 1995. Fortunately, the experimenters are in Goddard's own Guidance and Control Branch (code 712) working in conjunction with the Spartan spacecraft builders in code 740. The attitude will be determined onboard the spacecraft, but the data will be recorded and be available after the flight for ground processing. This will be the first opportunity to validate the proposed implementation

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of this capability in the institutional ground based attitude determination system used in the FDD.

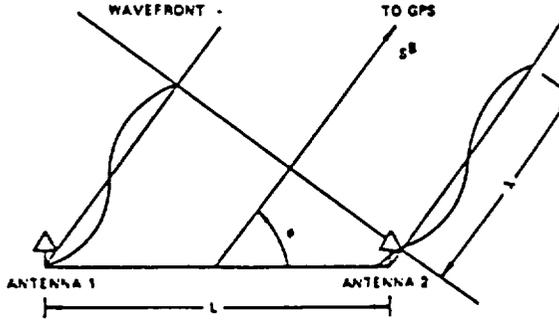


Figure 1

GPS Attitude Determination Geometry

Figure 1 illustrates the basic concept of determining a rotation angle from phase difference. The fundamental equation, which relates the phase difference received from a GPS satellite to the cosine of the angle between the baseline and line of sight to the GPS satellite, is given by

$$\cos \alpha = (n + k\phi) (\lambda / b) \quad (\text{Equation 1})$$

where: α = angle between the baseline and line of sight to the GPS spacecraft
 n = integer number of cycles in the phase difference between receivers
 ϕ = decimal part of the phase difference received from the GPS signal
 k = scale factor which depends on ϕ 's units
 λ = wavelength of the GPS signal (GPS has two frequencies, the L1 at 1575.42 MHz., and the L2 at 1227.6 MHz. The wavelengths are 0.19042541 meters and 0.24437928 meters, respectively)
 b = baseline length (for the Spartan spacecraft $b = 1.0$ meters)

With a pair of preferably orthogonal baselines it is possible to determine a line of sight vector to the GPS spacecraft. The above equation relates the direction cosines to the phase differences as follows:

$$\cos \alpha = (n_1 + k\phi_1) (\lambda/b) \quad (\text{Equation 2})$$

$$\cos \beta = (n_2 + k\phi_2) (\lambda/b) \quad (\text{Equation 3})$$

$$\cos \gamma = [1 - \cos^2 \alpha - \cos^2 \beta]^{1/2} \quad (\text{Equation 4})$$

These define a unit vector in the receiver coordinate system defined by the two orthogonal receiver

baselines fixed in the spacecraft and, therefore, the body coordinate system frame.

$$\begin{aligned} x_r &= \cos \alpha \\ y_r &= \cos \beta \\ z_r &= \cos \gamma \end{aligned}$$

If the receiver coordinate system is not coaligned with the body coordinate system then the unit vector is converted to the body coordinate system by the following:

$$\hat{x}_b = M^T \hat{x}_r \quad (\text{Equation 5})$$

where: M = rotation matrix which takes the body coordinate system (BCS) to the receiver system. The superscript indicates a transpose.

\hat{x}_r = unit vector to the GPS spacecraft in receiver coordinates and is defined as $[x_r, y_r, z_r]^T$

\hat{x}_b = unit vector to the GPS spacecraft in BCS

By combining this observed unit vector with the GPS position unit vector, obtained from the GPS receiver directly or analytically from previous ground processing, the FDD institutional attitude determination system (ADS) can use GPS data in the same manner as it currently uses star tracker data.

The only term in equation 1 that is still unknown is n , the integer number of cycles in the phase difference. Because some receivers measure only the fractional part of the phase difference, while others begin counting cycles at randomly large negative numbers for each locked signal so that a difference in carrier phase between antennas contains a meaningless number of whole cycles, the true integer number of cycles, n , between antenna measurements is unknown. Although there are several analytical search algorithms to resolve this ambiguity, the fact that the attitudes in this case are being determined on the ground in non-real-time we have an advantage of using a coarse idea of the attitude to determine n up front.

Integration of GPS Observations into Institutional Systems

OPERATIONS SYSTEMS

MTASS

The FDD developed a generic attitude ground support system from the software developed for the Extreme Ultra Violet Explorer (EUVE) and the Upper Atmospheric Research Satellite (UARS) missions. The motivation was to reduce errors as well as development, testing, and maintenance costs by having functions common to many missions contained within a single system. The ground support system is the Multi-mission Three-Axis Stabilized Spacecraft (MTASS) system. Since MTASS does not provide all the ground support functions needed to support each mission each mission requires some unique modules, for instance a telemetry processing system. However, MTASS does provide several generic methods to determine attitudes. These attitude determination functions are what will be addressed in subsequent sections for adaptation to using GPS observation data. The impact to the MTASS system should be minimal, with the only minor changes required for the introduction of the new GPS signal sensor. The telemetry processing as described above to turn the phase differences into observation vectors and pairing with reference vectors will be done upfront in the mission specific telemetry processing function.

Determine Real-time MACS Attitude

The real-time Modular Attitude Control System (MACS) attitude is determined using observations from a particular telemetry time frame, which is normally referred to single-frame attitude determination. The solution to the single-frame attitude problem comes from minimizing the loss function:

$$J(A) = \sum_{i=1}^n w_i \left| \hat{u}_B^i - A \hat{u}_R^i \right|^2 \quad (\text{Equation 6})$$

where: \hat{u}_B^i = observations unit vectors in the body coordinate system
 \hat{u}_R^i = reference unit vectors in the reference coordinate system
 w = weights applied to each observation/reference pairing

A = attitude matrix that relates the two coordinate systems

The popular method for finding the A which minimizes the above equation is suggested by Shuster (reference 2) and involves finding a maximum eigenvector to a modified version of the loss function:

$$J(q) = q^T K q \quad (\text{Equation 7})$$

where: q = 4 element quaternion which represents the transformation between the body and reference coordinate systems; 3 elements are associated with direction, the 4th with the magnitude with the relationship: $1 = (q_1^2 + q_2^2 + q_3^2 + q_4^2)^{1/2}$
 K = 4x4 matrix derived from the observation vectors, reference vectors and weights (see reference 2 for definition)

The adaptation of GPS observations involves converting the phase differences for a particular receiver baseline into an observation vector and using a reference ephemeris for each GPS pairing them with a reference vector.

All of the changes to the current real-time system are made up front in the processing of the telemetry into engineering data (observation vectors). The user supplied parameters needed for the this telemetry processing are listed below:

- o uncertainty for the GPS position vector
- o number of possible GPS spacecraft visible at each time point
- o number of GPS sensors
- o Frequency of carrier signal
- o scale factor to convert phase difference from telemetry into a decimal number
- o baseline length

Determine Non-real-time MACS Attitude

MTASS also provides for an off-line, non-real-time attitude determination function for a better estimate of state by using batches of multiple observations and propagating those to a common (epoch) time. The procedure minimizes the loss function given by:

$$J = \frac{1}{2} ([\hat{u}] - [\hat{w}])^T W ([\hat{u}] - [\hat{w}]) \quad (\text{Equation 8})$$

where: $[\hat{u}]$ = set of observations in the body coordinate system
 $[\hat{w}]$ = set of reference (or observation model) vectors
 W = symmetric, non-negative definite matrix which weights the individual contributions of each observation or reference pair

The method used in MTASS to find the state that minimizes the above function is the batch least-squares. This method estimates a state (in our case the attitude estimate) at a particular epoch based on the a priori knowledge and sensor observations. Attitude estimates from this time point are determined by propagation using rate information.

Adapting GPS observations to the batch least-squares method is identical to the processing involved in the real-time processing defined above. The only difference is the collection of observation/reference vector pairs to be applied at a single epoch time. The weights for each of these pairs which are inversely proportional to the expected accuracy of the measurement are placed along the diagonal of the W matrix. User input parameters are similar to those for the real-time attitude determination function listed above.

Sequential Estimation of Attitude

Another method employed by MTASS is the sequential processing of observations and reference vectors, like the real-time method, but including modeling of the state and dynamics noise to improve the estimate. Batches of observations need not be stored as the "information" from all previous measurements, declining in importance over time, is propagated in the form of the covariance matrix. This method uses a Kalman Filter, which has become increasingly popular with the advances in computer technology to allow faster processing.

The equations for the Kalman Filter differ only in the type of filter employed (standard, linear or extended Kalman Filter) and the modeling for the measurement and dynamics, which depend on the state and desired estimation accuracy. Numerous papers have described implementations real-time sequential attitude estimation using GPS observations, see references 4 - 6 for some examples. Since MTASS already converts the phase differences into observation vectors in the body coordinate frame, the Kalman Filter measurement model can compute the estimated state (at this time only attitude, but will be extended to include biases and misalignments in the state at a later date) much like star tracker observations.

This Kalman Filter is an extended filter, due to its non-linearities in the differential equations, is linearized about the latest estimate of the state. Now the state being estimated is not the actual attitude, but the attitude error. At each measurement update, the state error is added to the state estimation; the state error is then reset. In between measurement updates, the state is propagated to the next step using rate information. The measurement matrix is comprised of the partial derivatives of the state differential equations with respect to each of the state error elements and evaluated with respect to the latest estimate of the state. This model for the estimation of attitude using GPS observations has been successfully tested using a realistic simulation of the GPS constellation, and a typical low-earth spacecraft with the modeling of the spacecraft dynamics.

For an 'improved' or definitive attitude solution, MTASS employs a Rauch-Tung-Streifel (RTS) backward smoother to augment the extended Kalman Filter equations. The RTS implementation can use up to twenty-four hours of data at a time. The philosophy behind the backward smoother is to make use of the knowledge of the state at the end of processing all the observation data and running backward in time to apply this knowledge to each time step, which improves the estimate. For a more detailed discussion of the RTS, and other smoothers, it is suggested that the reader find a book on optimal estimation techniques (such as reference 3).

ANALYSIS SYSTEMS

ADEAS

The Attitude Determination Error Analysis System (ADEAS) is a general-purpose linear error analysis tool for spacecraft attitude determination. ADEAS does not process sensor data but simulates the attitude determination logic and computes the resulting attitude determination accuracy. The spacecraft attitude determination scenarios that can be analyzed by ADEAS are described below:

- o from low-altitude Earth orbits to International Sun-Earth Explorer (ISEE) -3 type of Earth-Sun libration point orbits
- o Spin-stabilized or three-axis-stabilized spacecraft attitudes
- o batch weighted-least-squares and sequential filter attitude determination methods

- o sensor complements, which are subsets of Sun sensor, Earth sensors, star sensor, gyros, magnetometers and now GPS receivers

ADEAS' strength lies in its flexibility: it was designed to include most of the existing and anticipated Earth satellite attitude determination systems. Individual error analysis programs no longer need to be written for each spacecraft as ADEAS allows an analyst to define any (low Earth) orbit and any attitude profile, with a specified set of corrupted sensors taking measurements at a defined sampling rate.

Given that an attitude determination process necessarily involves errors—e.g., measurement noise, sensor misalignments, gyro drifts—it is important to understand and evaluate how an estimate of the spacecraft attitude is affected by the presence of such errors. ADEAS allows an analyst to specify the type and magnitude of these errors for a particular configuration and computes the resulting uncertainties in a user-specified subset of measurement and dynamic parameters. These errors can be either “solved for” or “considered,” depending on how these errors will be handled operationally; the user in effect can, through using ADEAS, assess the merits of including (or not) certain errors as states and/or solving for the errors in some other way operationally.

Adding GPS models to ADEAS not only provides a method of determining attitude errors as driven by mission unique error sources (e.g. misalignments of the baselines due to antennas mounted on deployables) but also allows the user to assess overall attitude uncertainties for systems that have additional sensors like gyroscopes or magnetometers. Fortunately, GPS observations can be modeled in ADEAS very much like the current sensors. The only exception is the observation vectors are determined from a model which produces phase differences for each GPS sensor, defined as a baseline containing a pair of GPS receivers. The model applies the expected components of the uncertainty in the measurements. The uncertainty in the measurements, or observations, is due to noise, biases and misalignments are modeled essentially as follows:

$$\Delta \bar{r} = AM(L + \delta L)\bar{e}_b + \bar{\beta} + \bar{v}$$

where: Δr = expected range difference
 A = rotation matrix from misaligned body frame to inertial space
 M = rotation matrix from body to misaligned body (solved for or considered)
 L = baseline length

δL = baseline length error (solved for or considered)
 e_b = vector from master to slave in body frame
 β = line bias vector (solved for or considered)
 v = noise vector

As ADEAS does not actually compute attitudes, only covariances, integer ambiguities need not be determined. The measurements are converted to observation unit vectors, as described earlier in this paper. An ephemeris file for each of the GPS satellite provides the reference unit vector modeling. From this information and user supplied parameters as to what is solved for and what is considered, as described above, a covariance analysis is done over a specified interval. For a more complete discussion on covariance analysis the reader is directed toward a book on optimal estimation (such as reference 3).

GPS Visibility Prediction Tool

To help in investigation of attitude determination using GPS observations, the second author wrote a visibility prediction software utility that gives the user flexibility in determining GPS observation times and statistics. This prediction utility provides insight into the number of observations that can be expected for a particular mission depending on such things as:

- o mission altitude
- o inclination
- o alignment of GPS receiver baselines
- o boresight of each receiver
- o GPS acquisition mask for the receivers

Currently the software utility executes on an IBM compatible PC and is written in Microsoft FORTRAN. The user can adjust the configuration of the GPS or user spacecraft and the GPS receivers by means of interactive menus. This allows for greater flexibility in setting up a specific scenario. The internal modeling makes use of a two-body propagator for each of the twenty four GPS spacecraft in the constellation and for the user spacecraft. Future plans include allowing the user to read in a more accurate ephemeris file generated by an outside source. The user spacecraft attitude is modeled by propagating the attitude state to the next time step using kinematic equations. This allows for a fairly good representation of how motion affects GPS visibility. The receivers are modeled as a baseline aligned in the body coordinate system. The receivers' boresights are modeled as vectors in the body coordinate system, with a user supplied mask angle, which represents a cone around the boresight in

which GPS satellites will be visible. Simple geometric equations take into account if each of the GPS satellite are within the specified mask and not occulted by the Earth. Figure 2 shows a typical plot of GPS observations for a low Earth orbit satellite.

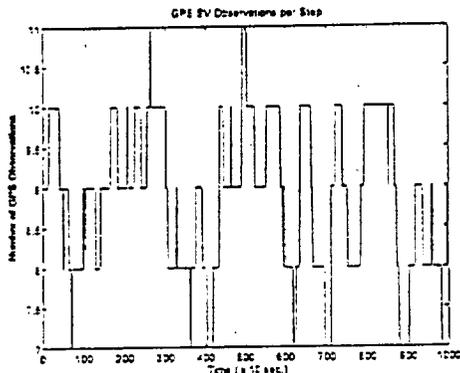


Figure 2

This utility outputs two concise report files. The first report file gives a step by step account of which individual GPS spacecraft are visible as well as the total number visible at any given time step. The second report gives a summary of the scenario's configuration, statistics for each GPS in the constellation on how often it was visible during the simulation and the percentage of time, a density table on the distribution of total GPS spacecraft visible, and the minimum and maximum time each GPS satellite is visible. The utility will be enhanced to predict the geometric dilution of precision, or GDOP, for a particular mission.

GPS Simulator

To test out the developed attitude estimation functions, a simulator would be needed to generate the GPS measurement data. A menu driven simulator was developed by the second author to execute on an IBM compatible PC and written in Microsoft FORTRAN. The simulator takes much of its modeling from the prediction utility. There were some major modifications to include more detailed and new models. The kinematics in the prediction utility were replaced by a more accurate dynamics model to allow for more precise modeling of the attitude. The GPS simulator takes the observations at each time step and processes them into realistic measurement data. The general steps involved in this process for each visible GPS at each time step are as follows:

1. Accept raw measurements, converted into whole and fractional parts of wavelengths. Imbedded in this step would be misalignments of each antenna. These are user supplied parameters.

2. Computing the phase difference for each baseline. An uncertainty is added to this quantity to account for the random properties such as noise. This is a user supplied quantity.
3. The integer portion of the phase difference is stripped off leaving only the decimal portion for processing.
4. The decimal portion of the phase difference is then adjusted for the least significant bit that may be found in the data word, that is the resolution of the data.

The next step in the simulation would be equivalent to the sensor processing normally found onboard in the flight software. The steps involved in this process are as follows for each GPS visible:

1. The input for each baseline, the final decimal phase difference, is input to a routine to determine the integer ambiguity.
2. Another routine monitors the change in phase difference to determine when the integer ambiguity needs to be updated.
3. Finally the phase differences from the two baselines are converted into observation unit vectors in the sensor coordinate system.
4. The observation unit vectors are paired with a reference vector based on GPS spacecraft id.

The output is then a set of n observation/reference unit vector pairs, where n is the number of GPS satellite visible at that time step. These pairs of data is then input into the desired attitude estimation function as was described earlier. Figures 3 and 4 show examples of the Kalman Filter and Q-method error in estimating the attitude using the GPS simulator.

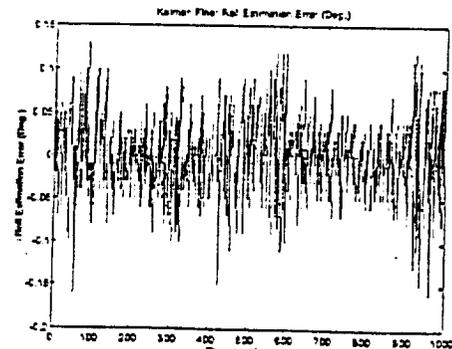


Figure 3

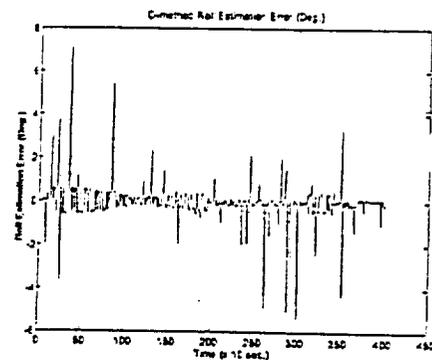


Figure 4

For more detailed discussion of the data analysis, see reference 7.

Future Work

Flight dynamics, along with operations support, traditionally performs analysis both for specific upcoming missions as well as examining existing flight data in order to improve the performance of attitude sensors and effectively save hardware costs for missions farther in the future. GPS, as it is so new, yet has created so much excitement in the era of end to end cost savings, proves fruitful for both realms of analysis.

GADACS data analysis

As stated above, the GADACS experiment is slated for launch in November 1995. As it was conceived and built at Goddard, the ability to properly process the data on the ground, checking FDD's GPS additions to the MTASS system in preparation for future support as well as provide independent verification of the experimenter's data, would be highly desirable.

TRACE data analysis

The Transition Region and Coronal Explorer (TRACE) is the 4th in the Small Explorer series and is slated to launch in September 1997. It will fly a GPS receiver, with antennas mounted on the backs of 3 separate solar panels. The FDD ground post processed spacecraft roll angle used by scientists to correlate to their data has a 1σ goal of 0.1° . This goal will be difficult to meet with the current hardware of a magnetometer and gyroscope. However, the GPS receiver should assist in nailing down this angle to within the goal.

The ground software to be used by TRACE will be a new modular workstation based system currently being developed. Filters similar to those for MTASS are to be implemented, as are the GPS models. The system will be able to take a quaternion output as well as raw carrier phase measurements.

The FDD hopes to provide an independent verification of the performance of the GPS receiver, one that has no space flight heritage, through several means. The scientists are providing (infrequently) several contiguous orbits of spacecraft roll angles derived from the science data. These solutions should be better than 0.1° , and are to be used to calibrate the magnetometer and the gyroscope. This data can also be used to check the GPS determined roll angle.

Experience shows, however, that hardware systems, especially those without heritage and with extensive software, should be checked thoroughly through examining of raw measurements, in this case carrier phase. By independent checking, onboard system hardware and software errors can be decoupled and closely examined. Also, calibration parameters, such as line biases or baseline misalignments can be corrected for, as is traditionally done for other sensors.

Finally, as these antennas are on separate deployables, the issue of baseline misalignment and length uncertainty are being closely examined for TRACE both through using ADEAS as modified above, and through other studies. Once inflight, this configuration should be quite interesting to the GPS attitude community at large for study as well as for future mission planning as it is not always convenient to place GPS antennas on certain shaped spacecraft, telescopes in particular.

Conclusions

The Goddard Flight Dynamics Division is currently preparing for a completely new sensor to begin flying experimentally in late 1995, and routinely in 1997. The FDD has updated models in the current operations support and analysis software, and is preparing models for new workstation based operations software to accommodate GPS measurements. Data from EUVE, although sparse for attitude by nature of the GPS configuration on the spacecraft, was examined. Finally, just as FDD has a long heritage of verifying and improving performance for traditional attitude sensors through examining flight data, preparations and studies are underway to support specific upcoming missions for GPS attitude determination.

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